Math 371 (Differential Equations), Section 3 Your name: KEY

Quiz 2

September 15, 2009

1. Calculate the derivative $\frac{du(x(t),y(t))}{dt}$, where $u(x,y) = y^x$, $x(t) = \ln(t)$ and $y(t) = t^2$ using the chain rule

Let $u : \mathbb{R}^2 \to \mathbb{R}$ $\quad (x,y) \mapsto u(x,y) = y^x$

$g : \mathbb{R} \to \mathbb{R}^2$ $\quad g(t) = \begin{pmatrix} \ln t \\ t^2 \end{pmatrix}$

$$\frac{du(x(t),y(t))}{dt} \underset{\text{chain rule}}{=} u'(g(t))\cdot g'(t) = \left[\frac{\partial u}{\partial x}(g(t)), \frac{\partial u}{\partial y}(g(t))\right]\cdot\begin{bmatrix}(\ln t)' \\ (t^2)'\end{bmatrix} =$$

$$= \left[\ln y \cdot y^x, (x)y^{x-1}\right]\Big|_{\substack{x=\ln t\\ y=t^2}} \cdot \begin{bmatrix}\frac{1}{t} \\ 2t\end{bmatrix} = \boxed{\ln t^2 \cdot (t^2)^{\ln t} \cdot \frac{1}{t} + \ln t \cdot (t^2)^{\ln t - 1} \cdot 2t}$$

2. Show that the following differential equation is exact and solve it

$$y\,dx + (x-y)dy = 0$$

$P(x,y) = y$, $Q(x,y) = x - y$

$\frac{\partial P}{\partial y} = 1 = \frac{\partial Q}{\partial x}$. $\Rightarrow$ equation is exact.

Find $u(x,y)$

$\frac{\partial u}{\partial x} = P(x,y) = y$

$u(x,y) = \int y\,dx = yx + k(y)$

$\left.\begin{array}{l}\frac{\partial u}{\partial y} = Q(x,y) = x - y \\ \quad x + k'(y)\end{array}\right\} \Rightarrow k'(y) = -y$ and $k(y) = -\frac{y^2}{2} + C$

Thus, final answer is

$u(x,y) = \boxed{yx - \frac{y^2}{2} = C}$