Math 371, Section 3 Your name: ____________________

Quiz 1

September 8, 2009

1. Solve the initial value problem $y' = 2y + 1, \quad y(0) = 3$

Solution: We have $\frac{dy}{dx} = 2y + 1$. First we check that $y(x) = -\frac{1}{2}$ is a solution. Now assume $y(x) \neq -\frac{1}{2}$ and thus we can divide by $2y + 1$.

$\dfrac{dy}{2y+1} = dx,$

$\displaystyle\int \frac{dy}{2y+1} = \int dx,$

$\dfrac{1}{2} \ln |2y + 1| = x + C$

$|2y + 1| = Ce^{2x}, C > 0$ (here we renamed the constant)

$2y + 1 = Ce^{2x}, C \neq 0$

$2y + 1 = Ce^{2x}$, $C \in \mathbb{R}$ (here we took into account the solution $y = -\frac{1}{2}$ that we have found on top)

Thus the general solution is $y = \frac{1}{2}(Ce^{2x} - 1)$. Now plug the initial value condition $y(0) = 3$:

$$3 = \frac{1}{2}(Ce^0 - 1) \quad \Rightarrow \quad C = 7.$$

Therefore the final answer is $y = \frac{1}{2}(7e^{2x} - 1)$.

2. Let $A = \begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 1 \\ 0 & 1 \\ 2 & 0 \end{pmatrix}$. Calculate $2A$ and AB.

Solution: $2A = 2 \cdot \begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 0 & -2 \\ 4 & 2 & 0 \end{pmatrix}$

$AB = \begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} -1 & 1 \\ 0 & 1 \\ 2 & 0 \end{pmatrix} =$

$\begin{pmatrix} 1 \cdot (-1) + 0 \cdot 0 + (-1) \cdot 2 & 1 \cdot 1 + 0 \cdot 1 + (-1) \cdot 0 \\ 2 \cdot (-1) + 1 \cdot 0 + 0 \cdot 2 & 2 \cdot 1 + 1 \cdot 1 + 0 \cdot 0 \end{pmatrix} = \begin{pmatrix} -3 & 1 \\ -2 & 3 \end{pmatrix}$

3. Find the eigenvalues of the matrix $\begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix}$.

Solution: The eigenvalues are the solutions of the equation

$\det \begin{pmatrix} 3 - \lambda & 1 \\ 2 & 2 - \lambda \end{pmatrix} = 0$

$(3 - \lambda)(2 - \lambda) - 2 \cdot 1 = \lambda^2 - 5\lambda + 4 = (\lambda - 1)(\lambda - 4) = 0.$

Thus, the eigenvalues are $\lambda = 1$ and $\lambda = 4$.