Math 304 (Linear Algebra), Section 1 Your name: KEY

Quiz 1

September 9, 2009

1. Determine if the following system of linear equations is consistent

$$\begin{cases} x_1 - 2x_2 + x_3 - 4x_4 = 1 \\ x_1 + 3x_2 + 7x_3 + 2x_4 = 2 \\ x_1 - 12x_2 - 11x_3 - 16x_4 = 5 \end{cases}$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & -4 & 1 \\ 1 & 3 & 7 & 2 & 2 \\ 1 & -12 & -11 & -16 & 5 \end{array}\right] \begin{matrix} r_2 \to r_2 - r_1 \\ \longrightarrow \\ r_3 \to r_3 - r_1 \end{matrix} \left[\begin{array}{cccc|c} 1 & -2 & 1 & -4 & 1 \\ 0 & 5 & 6 & 6 & 1 \\ 0 & -10 & -12 & -12 & 4 \end{array}\right] \xrightarrow{r_3 \to r_3 + 2r_2}$$

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & -4 & 1 \\ 0 & 5 & 6 & 6 & 1 \\ 0 & 0 & 0 & 0 & 6 \end{array}\right]$$

↑ pivot column.

The last column is pivot $\Rightarrow$ the system is inconsistent

2. Consider the system of linear equations

$$\begin{cases} 5x_1 - 2x_2 + 6x_3 = 0 \\ -2x_1 + x_2 + 3x_3 = 1 \end{cases}$$

(a) Reduce the augmented matrix A of the system to the reduced row echelon form

(b) What are the pivot columns in A?

(c) What is the rank of A?

(d) Solve the system completely

$$\left[\begin{array}{ccc|c} 5 & -2 & 6 & 0 \\ -2 & 1 & 3 & 1 \end{array}\right] \xrightarrow{r_1 \to r_1 + 2r_2} \left[\begin{array}{ccc|c} 1 & 0 & 12 & 2 \\ -2 & 1 & 3 & 1 \end{array}\right] \xrightarrow{r_2 \to r_2 + 2r_1}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 12 & 2 \\ 0 & 1 & 27 & 5 \end{array}\right] \leftarrow \text{RREF}$$

(a) ↗

(b) Pivot columns are the first and the second: $\begin{pmatrix} 5 \\ -2 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \end{pmatrix}$

(c) rank = 2

(d) $\boxed{\begin{array}{l} x_1 = 2 - 12x_3 \\ x_2 = 5 - 27x_3 \\ x_3 = \text{anything} \end{array}}$